



Disaggregating Supply and Use Tables using a Generalized, Three-Steps RAS: an application to the SUT of Galicia.

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Introduction

WHY DISAGGREGATING SUT MATTERS?

- When comparing two different economies through their Supply and Use tables, it is convenient to have matrix that share a common product-by-industry classification.
- In order to achieve this common classification, aggregated models are built.
- This way, relevant information about the structure of the economies we ought to analyze can be lost.
- It is then important to find a methodology for disaggregation that can maximize the available information while ensuring the final coherence and economic meaning of the resulting SUT system.



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The problem to be solved

MATHEMATICAL ANALYSIS

Common aspects for Supply and Use Tables

- Problem to be solved:

Transforming an aggregated $p \times q$ matrix (Z^A) into a disaggregated $m \times n$ matrix (Z^*) taking as a reference a system described in a $m \times n$ matrix (Z^W).

- In this context:

- $p < m$
- $q < n$.

Wolsky's two-stages procedure for disaggregation

- In the first stage of the process, an “augmented matrix” is built. In this matrix, new rows and columns are introduced by splitting cells considering constant weights (w_i). As a result, new industries are introduced in the economy with the same technological characteristics.
- After that, a “distinguishing matrix” is introduced. In this matrix the technological differences between the disaggregated industries are considered through specific parameters (σ, δ, ξ).
- The new disaggregated matrix is finally obtained by adding the augmented and the distinguishing matrixes.

$$\mathbf{A} = \begin{pmatrix} 0.02 & 0.05 & 0.05 & 0.05 \\ 0.35 & 0.10 & 0.45 & 0.45 \\ w_1 \times 0.08 & w_1 \times 0.25 & w_1 \times 0.15 & w_1 \times 0.15 \\ w_2 \times 0.08 & w_2 \times 0.25 & w_2 \times 0.15 & w_2 \times 0.15 \end{pmatrix} + \begin{pmatrix} 0 & 0 & w_2 \delta_1 & -w_1 \delta_1 \\ 0 & 0 & w_2 \delta_2 & -w_1 \delta_2 \\ \sigma_1 & \sigma_2 & (\frac{1}{2} \delta_3 + \xi) w_2 + \sigma_3 & -(\frac{1}{2} \delta_3 + \xi) w_1 + \sigma_3 \\ -\sigma_1 & -\sigma_2 & (\frac{1}{2} \delta_3 - \xi) w_2 - \sigma_3 & -(\frac{1}{2} \delta_3 - \xi) w_1 - \sigma_3 \end{pmatrix}.$$

The first term on the right-hand side is the augmented matrix and the second term is the distinguishing matrix.

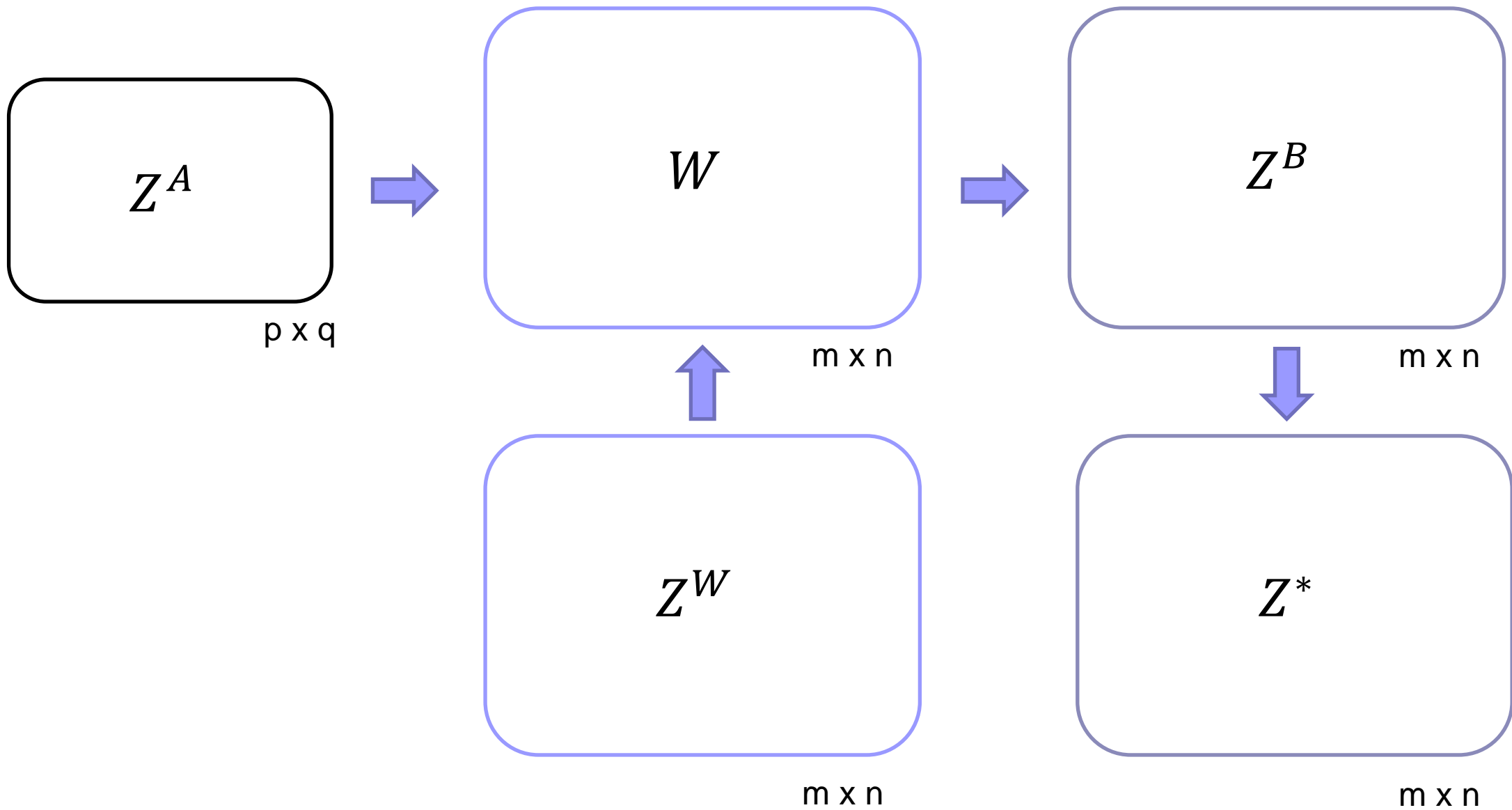
$$\begin{aligned}
 -0.06 &\leq \sigma_1 \\
 -0.19 &\leq \sigma_2 \\
 -0.11 &\leq \sigma_3
 \end{aligned}$$

Wolsky's Disaggregation procedure: an illustration.

Source: Wolsky (1984, p. 288).

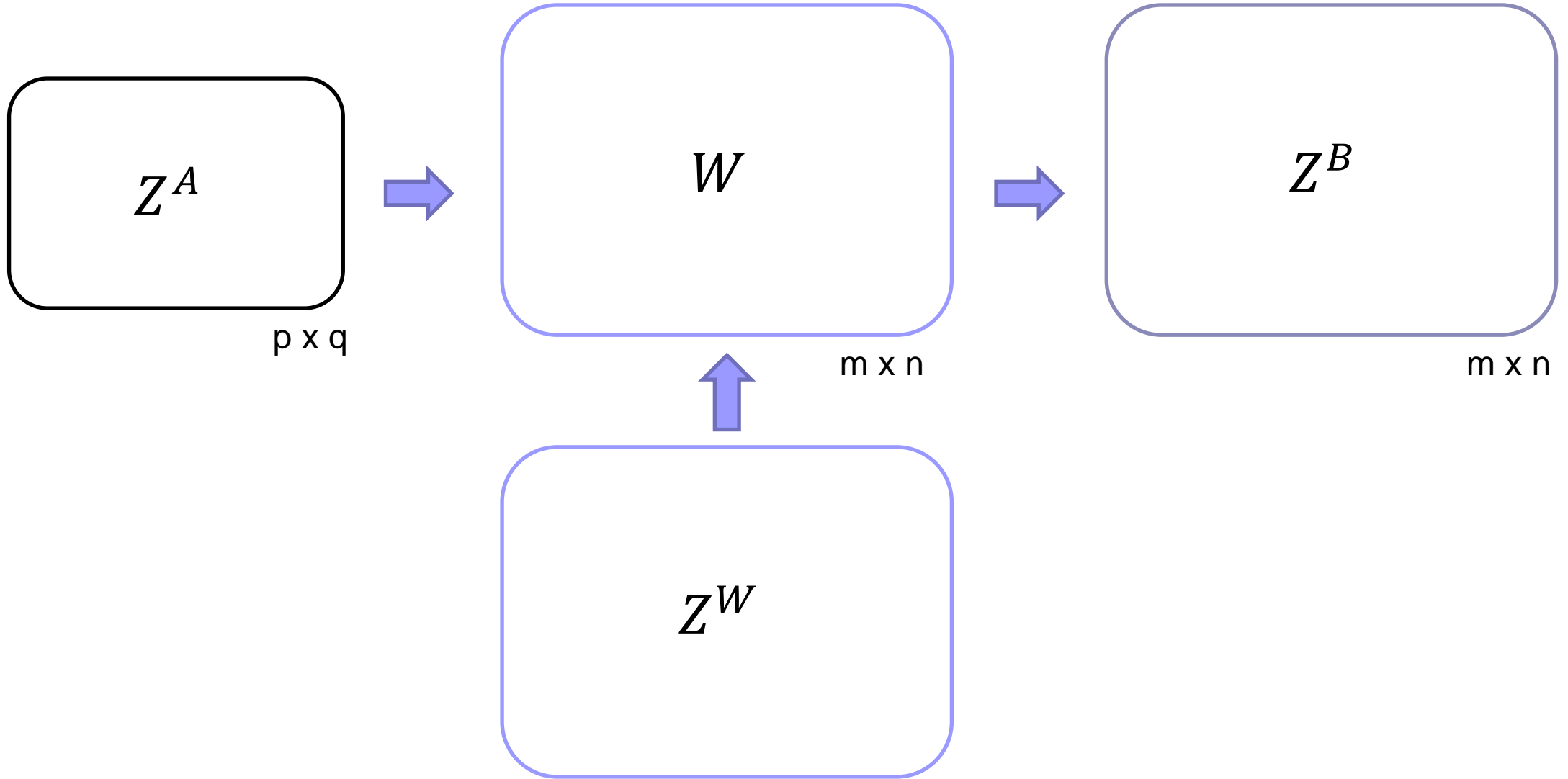
Our two-stages procedure

- In the first stage, we build an augmented matrix (Z^B) using a matrix of weights (W). These weights are taken from a system that we take as a reference (Z^W) or from other sources of information.
- At this point, the technologies of the new disaggregated products or industries are not necessarily equal anymore.
- In a second stage, we adjust the augmented matrix to other data we might have about the row and column totals, maximizing the use of available information. The result is an adjusted matrix (Z^*).



The augmented matrix: previous considerations

- Two inputs are needed to build the augmented matrix Z^B .
 1. On the one hand, the z_{ij}^A elements of the aggregated matrix Z^A .
 2. On the other hand, the weights w_{ij} contained in matrix W . These elements are established on the basis of a system that is taken as reference (Z^W).
- For explanatory purposes, let the row p and the column q of Z^A the ones to be disaggregated into the rows $m - 1$ and m and the columns $n - 1$ and n in matrix Z^B .



The construction of the weights

- For the new rows $m - 1$ and m :

- $$W_{m-1,j} = \frac{z_{m-1,j}^W}{(z_{m-1,j}^W + z_{m,j}^W)}$$

- $$W_{m,j} = \frac{z_{m,j}^W}{(z_{m-1,j}^W + z_{m,j}^W)}$$

- $\forall j = 1, \dots, n - 2$

- For the new columns $n - 1$ and n :

- $$W_{i,n-1} = \frac{z_{i,n-1}^W}{(z_{i,n-1}^W + z_{i,n}^W)}$$

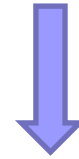
- $$W_{i,n} = \frac{z_{i,n}^W}{(z_{i,n}^W + z_{i,n+1}^W)}$$

- $\forall i = 1, \dots, m - 2$

- In the intersections between disaggregated rows and columns contradictory information can appear. To solve this problem, we establish specific weights, equivalent to those ξ in Wolsky's procedure.

- $w_{m-1,n-1} = z_{m-1,n-1}^W / (z_{m-1,n-1}^W + z_{m,n-1}^W + z_{m-1,n}^W + z_{mn}^W)$
- $w_{m,n-1} = z_{m,n-1}^W / (z_{m-1,n-1}^W + z_{m,n-1}^W + z_{m-1,n}^W + z_{mn}^W)$
- $w_{m-1,n} = z_{m-1,n}^W / (z_{m-1,n-1}^W + z_{m,n-1}^W + z_{m-1,n}^W + z_{mn}^W)$
- $w_{mn} = z_{mn}^W / (z_{m-1,n-1}^W + z_{m,n-1}^W + z_{m-1,n}^W + z_{mn}^W)$

$$\begin{pmatrix} z_{m-1,n-1}^W & z_{m-1,n}^W \\ z_{m,n-1}^W & z_{mn}^W \end{pmatrix}$$



$$\begin{pmatrix} w_{m-1,n-1} & w_{m-1,n} \\ w_{m,n-1} & w_{mn} \end{pmatrix}$$

The W matrix in our example:

$$W = \begin{pmatrix} 1 & \dots & W_{n-1,1} & W_{n1} \\ \vdots & \ddots & \vdots & \vdots \\ W_{m-1,1} & \dots & W_{m-1,n-1} & W_{m-1,n} \\ W_{m1} & \dots & W_{m,n-1} & W_{mn} \end{pmatrix}$$

Building matrix Z^B

- For non-disaggregated cells:

$$z_{ij}^B = z_{ij}^A$$

- In the case of Z^B elements:

$$\forall i = 1, \dots, m - 2$$

$$\forall j = 1, \dots, n - 2$$

- In the case of Z^A elements:

$$\forall i = 1, \dots, p - 1$$

$$\forall j = 1, \dots, q - 1$$

$$(z_{p1}^A \quad \cdots \quad z_{p,q-1}^A)$$

New rows



$$\begin{pmatrix} z_{p1}^A W_{m-1,1} & \cdots & z_{p,q-1}^A W_{m-1,n-2} \\ z_{p1}^A W_{m1} & \cdots & z_{p,q-1}^A W_{m,n-2} \end{pmatrix}$$

$$\begin{pmatrix} z_{1q}^A \\ \vdots \\ z_{p-1,q}^A \end{pmatrix}$$

New columns



$$\begin{pmatrix} z_{1q}^A W_{1,n-1} & z_{1q}^A W_{1n} \\ \vdots & \vdots \\ z_{p-1,q}^A W_{m-2,n-1} & z_{p-1,q}^A W_{m-2,n} \end{pmatrix}$$

Intersection
between new
rows and columns

$$(z_{pq}^A)$$



$$\begin{pmatrix} z_{pq}^A W_{m-1,n-1} & z_{pq}^A W_{m-1,n} \\ z_{pq}^A W_{m,n-1} & z_{pq}^A W_{mn} \end{pmatrix}$$

Matrix Z^B in our example:

$$Z^B = \begin{pmatrix} z_{11}^A & \cdots & z_{1q}^A w_{1,n-1} & z_{1q}^A w_{1n} \\ \vdots & \ddots & \vdots & \vdots \\ z_{p1}^A w_{m-1,1} & \cdots & z_{pq}^A w_{m-1,n-1} & z_{pq}^A w_{m-1,n} \\ z_{p1}^A w_{m1} & \cdots & z_{pq}^A w_{m,n-1} & z_{pq}^A w_{mn} \end{pmatrix}$$

The adjusted matrix

- The augmented matrix Z^B obtained after the first stage of the procedure has $m \times n$ dimensions and the new rows and columns present their own technology (borrowed from the matrix Z^W).
- However, the sum of the rows and columns of matrix Z^B might not be coincident with the information we might have about the marginal elements of matrix Z^* .
- Therefore, some adjustments must be done.



Constraints to be considered

1. The total sum of each row
2. The total sum of each column
3. The sum of the submatrices formed with the disaggregation of cells.

Row (1) and column (2) adjustments

- The two first adjustments to be considered can be solve through Iterative Proportional Fitting (IPF) procedures. In Input-Output economics, the RAS Method works as a particular case of IPF procedures.
- Bishop, Fienberg and Holland (1974); Macgill (1977) point to a series of necessary conditions for IPF algorithms to work. Two of theses conditions are particularly problematic when applying RAS-type methods to SUT:
 1. The existence of negative elements in the matrixes to adjust.
 2. A high proportion of null elements.

The solution for the negative elements

- In order to fulfil constraints (1) and (2) in a context with negative elements, the GRAS method introduced by Junius and Oosterhaven (2003) is used.
- Following this contribution, we split Z^B into two matrixes: P and N . P contains the positive elements of Z^B . N contains the absolute values of the negative elements of Z^B . Thus:

$$Z^B = P - N$$

The Generalized RAS (GRAS)

- Let \bar{z}^* and \underline{z}^* be the row and column vectors set to be true information. Let e be the sum vector with the appropriate dimensions. The solution of the GRAS problem has the following form:

$$(\hat{r}P\hat{s} - \hat{r}^{-1}N\hat{s}^{-1})e = \bar{z}^*$$

$$e(\hat{r}P\hat{s} - \hat{r}^{-1}N\hat{s}^{-1}) = \underline{z}^*$$

A solution for the high proportion of null elements and contradictory information: a work in progress

- In the rows and columns with only one adjustable element, we modify the GRAS procedure letting that lonely element to adjust itself only to its correspondent row or column.
- If a main production of an industry represents a too big proportion both in its row and column, then this element can be excluded from the GRAS procedure in order to facilitate convergence.
- In the case that the high number of non-adjustable elements makes contradictory information irreconcilable, it is possible to modify the constraints in some rows (\bar{z}_i^*) and columns (\underline{z}_j^*) in a similar sense as KRAS proposed by Lenzen et al. (2009).

Row-and-Column adjustment steps (1) & (2)

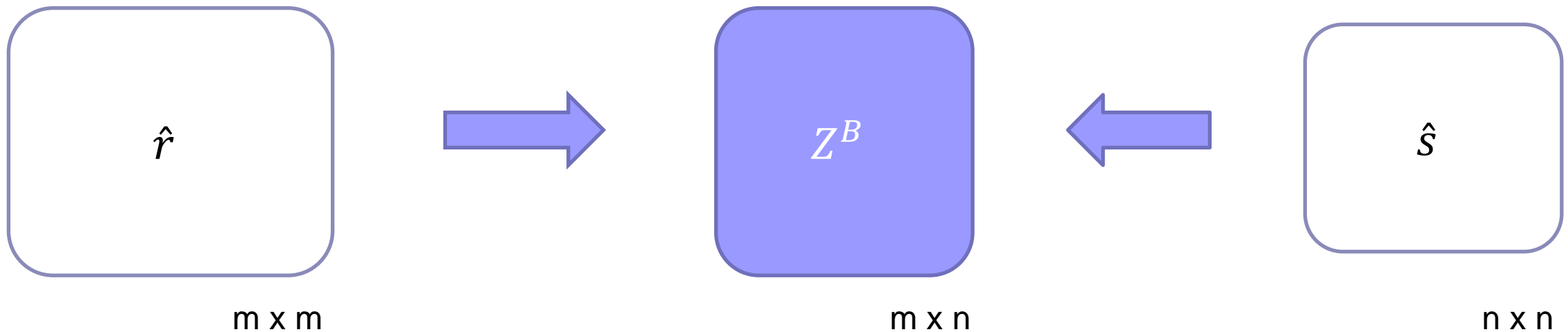
- Once the arrangements are made, the GRAS method is applied obtaining a new matrix $\hat{r}Z^B\hat{s}$.
- In each row-and-column adjustment, the GRAS algorithm is applied till the table converges to a solution where:

$$\max \left[\bar{v}_i^*(\tau) - \sum_j v_{ij}^B(\tau) \right] < |\varepsilon(\tau)|$$

$$\max \left[\underline{v}_j^*(\tau) - \sum_i v_{ij}^B(\tau) \right] < |\varepsilon(\tau)|$$

- Letting $\varepsilon(\tau)$ be a value as reduced as wanted and that can vary through the successive iterations.

- The elements of the diagonal matrixes \hat{r} and \hat{s} can be interpreted as first approximations to the technological differences between the adjusted matrix Z^* and the reference matrix Z^W .



Submatrix adjustments (3)

- When we introduce the adjustments for the row and column totals, restriction (3) is longer fulfilled. It is then necessary to introduce a third step in the adjustment process. In this case, we follow Gilchrist & St. Louis (1999, 2004).
- Let \mathcal{Z} be any block of new disaggregated cells. In our example, let \mathcal{Z}_{pq} be the block of cells generated from the disaggregation of z_{pq}^A . The submatrix adjustments are done applying the following coefficients:

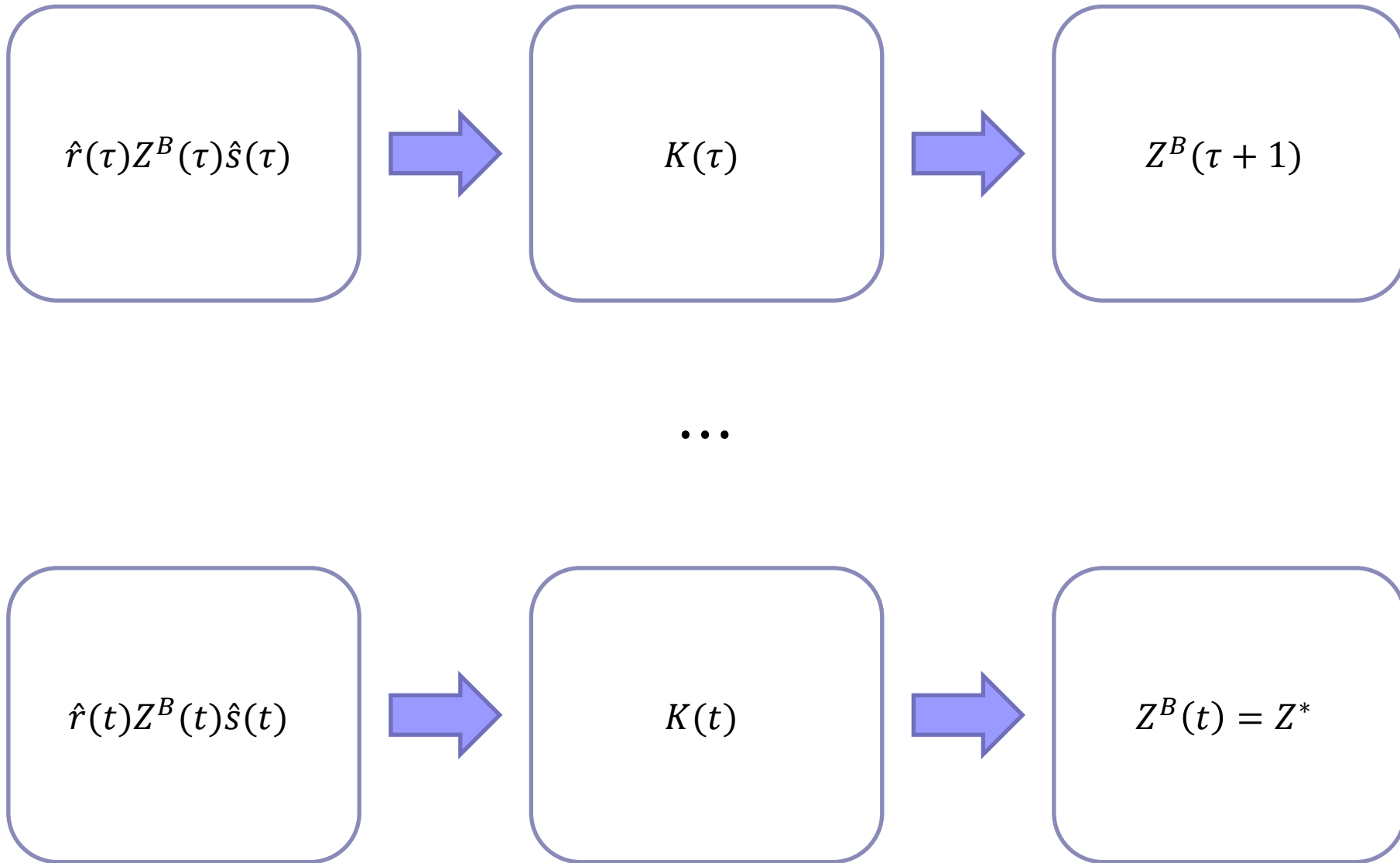
$$k_{ij} = z_{pq}^A / [(r_{m-1} z_{m-1,n-1}^B s_{n-1}) + (r_m z_{m,n-1}^B s_{n-1}) + (r_{m-1} z_{m-1,n}^B s_n) + (r_m z_{mn}^B s_n)]$$

The iterative process (1), (2) & (3)

- In this context, let an iteration be completed when the three steps of the process are completed.
- The result of an iteration is the base for the following one.
- Let $\tau = 0, 1, \dots, t$ be the iterations of the process:

$$Z^B(\tau + 1) = K(\tau) \circ [\hat{r}(\tau)Z^B(\tau)\hat{s}(\tau)]$$

- Note that in iteration $\tau = 0$ we take as a starting point the first augmented matrix $Z^B(0)$ resulting from the disaggregation of Z^A using the reference of Z^W .





Supply Tables and Use Tables

SPECIFIC ISSUES CONCERNING THE ADJUSTMENT PROCESS

Supply Tables (V): problems with the row and column totals

- If data about the supply by products and production by industries is available in the desired level of aggregation, then these values are established as targets in the first two steps of the adjustment procedure. So, for each iteration $\tau = 0, 1, \dots, t$ we have:

$$\bar{v}^*(0) = \dots = \bar{v}^*(t)$$

$$\underline{v}^*(0) = \dots = \underline{v}^*(t)$$

- Normally, this vector are unknown when we disaggregate SUT. For theses cases, an estimation procedure is introduced.

The known elements of $\bar{v}^*(\tau)$ and $\underline{v}^*(\tau)$

Let V^A the aggregated Supply Table and $V^B(\tau)$ the augmented Supply Table in iteration τ . If we keep the terms of our previous example it is guaranteed that:

$$\sum_j v_{ij}^B(\tau) = \sum_j v_{ij}^A$$

- For $V^B(\tau)$ rows:

$$\forall i = 1, \dots, m - 2$$

- For V^A rows:

$$\forall i = 1, \dots, p - 1$$

$$\sum_i v_{ij}^B(\tau) = \sum_i v_{ij}^A$$

- For $V^B(\tau)$ columns:

$$\forall j = 1, \dots, n - 2$$

- For V^A columns:

$$\forall j = 1, \dots, q - 1$$

Two sources of information for the disaggregated rows and columns marginal elements

- From the reference table V^W we obtain vectors \bar{v}^{EST} and \underline{v}^{EST} .
- For the new disaggregated rows:

$$\bar{v}_{m-1}^{EST} = \sum_j v_{pj}^A w_{m-1}$$

$$\bar{v}_m^{EST} = \sum_j v_{pj}^A w_m$$

- For the new disaggregated columns :

$$\underline{v}_{n-1}^{EST} = \sum_i v_{in}^A w_{n-1}$$

$$\underline{v}_n^{EST} = \sum_i v_{in}^A w_n$$

- From the augmented matrix $V^B(\tau)$ we obtain vectors $\bar{v}^{SUM}(\tau)$ and $\underline{v}^{SUM}(\tau)$.
- For the new disaggregated rows:

$$\bar{v}_{m-1}^{SUM}(\tau) = \sum_j v_{m-1,j}^B(\tau)$$

$$\bar{v}_m^{SUM}(\tau) = \sum_j v_{mj}^B(\tau)$$

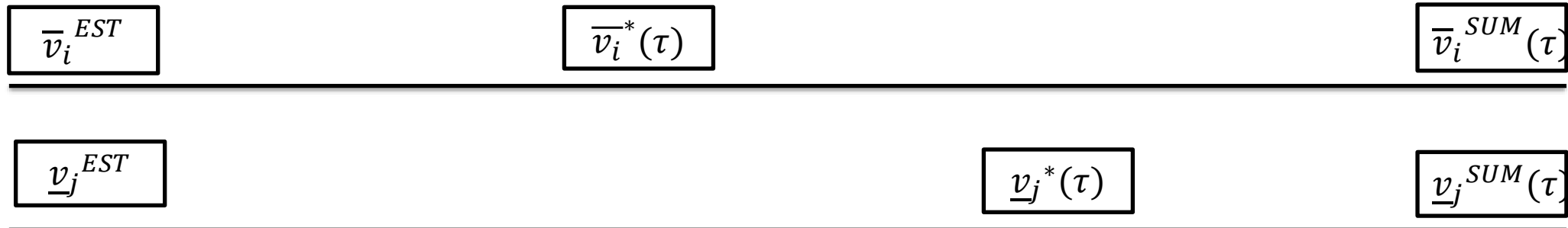
- For the new disaggregated columns:

$$\underline{v}_{n-1}^{SUM}(\tau) = \sum_j v_{n-1,j}^B(\tau)$$

$$\underline{v}_n^{SUM}(\tau) = \sum_j v_{nj}^B(\tau)$$

Estimating the unknown elements of target vectors $\bar{v}^*(\tau)$ and $\underline{v}^*(\tau)$

- Thus, two pairs of vector \bar{v}^{EST} and $\bar{v}^{SUM}(\tau)$ e \underline{v}^{EST} e $\underline{v}^{SUM}(\tau)$ are to be considered.
- We assume that the true value of the target vectors $\bar{v}^*(\tau)$ e $\underline{v}^*(\tau)$ stands somewhere in the middle of both values in a similar way as explored by Dalgaard & Gysting (2004).
- This way, the unknown elements of vectors $\bar{v}^*(\tau)$ and $\underline{v}^*(\tau)$ can be written as convex combinations of the elements in vectors $\bar{v}^{EST}(\tau)$; $\bar{v}^{SUM}(\tau)$ and $\underline{v}^{EST}(\tau)$; $\underline{v}^{SUM}(\tau)$ respectively.



The convex combination

- The convex combinations for each row and column is stated as it follows:

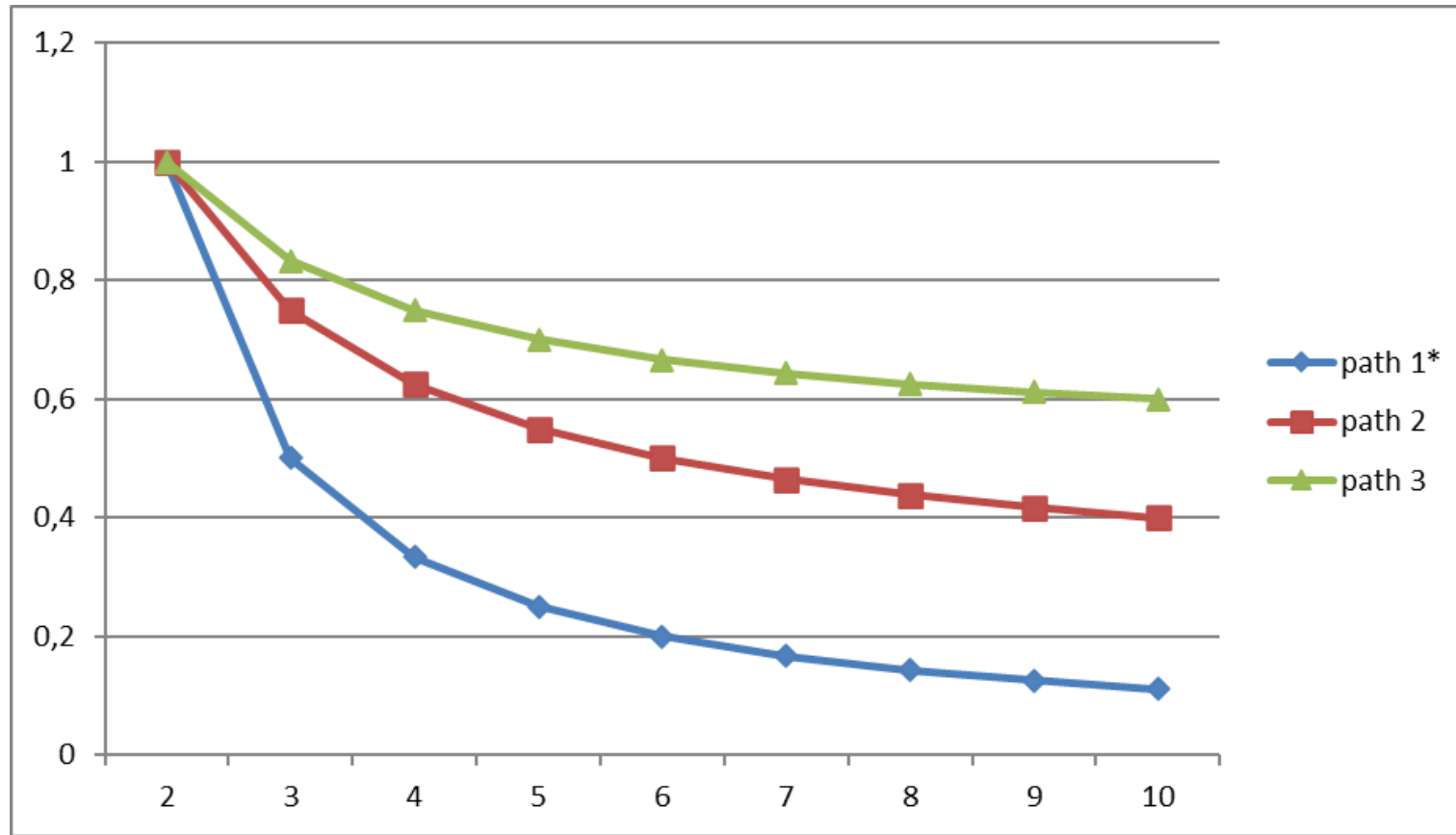
$$\bar{v}_i^*(\tau) = \alpha_i * \bar{v}_i^{EST}(\tau) + (1 - \alpha_i)\bar{v}_i^{SUM}(\tau)$$

$$\underline{v}_j^*(\tau) = \beta_j * \underline{v}_j^{EST}(\tau) + (1 - \beta_j)\underline{v}_j^{SUM}(\tau)$$

$$0 \leq \alpha_i \leq 1$$

$$0 \leq \beta_j \leq 1$$

- α_i and β_j operate as confidence factors in the information provided view the matrix taken as reference.



Possible paths for the α_i and β_j depending on the number of new rows or columns disaggregated from a prior one.

Source: Own elaboration.

The iterative process when estimating row and column totals in the Supply Table

- In this case, the iteration process has the same form as stated generally before:

$$V^B(\tau + 1) = K(\tau) \circ [\hat{r}(\tau)V^B(\tau)\hat{s}(\tau)]$$

- But, in iteration $\tau + 1$, the vectors \bar{v}^* and \underline{v}^* are calculated as convex combinations between $\bar{v}^*(\tau)$; $\underline{v}^*(\tau)$ and the row and column totals of matrix $V^B(\tau + 1)$:

$$\bar{v}_i^*(\tau + 1) = \alpha_i \bar{v}_i^*(\tau) + (1 - \alpha_i) \bar{v}_i^{SUM}(\tau + 1)$$

$$\underline{v}_j^*(\tau + 1) = \beta_j \underline{v}_j^*(\tau) + (1 - \beta_j) \underline{v}_j^{SUM}(\tau + 1)$$

$$0 \leq \alpha_i \leq 1$$

$$0 \leq \beta_j \leq 1$$

The Use Table (U)

- For the construction of the Use Table, the procedure is similar to the one employed for the Supply Table.
- In first place, we disaggregate a matrix U^A using weights taken from a reference matrix U^W obtaining an augmented matrix U^B .
- Secondly, the adjustment process begins, taking into account restrictions for the rows, columns and submatrices of the table.
- The difference between the Supply and the Use Tables is that in the later we already know the target values for the marginal elements of the matrix. As a consequence, no estimation process is needed.

Target vectors $\bar{u}^*(\tau)$ and $\underline{u}^*(\tau)$

- For the SUT to be coherent, it is necessary that the total supply of each product equals its total destinations. This implies, in the context of our problem, that:

$$\bar{u}^* = \bar{v}^*$$

- Analogously, the production by industries must be the same in both Supply and Use Tables. This implies that:

$$\underline{u}^* = \underline{v}^*$$

- Since $\bar{v}^*(t)$ and $\underline{v}^*(t)$ are considered to be true information at the end of the Supply Table's construction process, in the case of the Use Table we have for each iteration τ :

$$\bar{u}^*(0) = \bar{u}^*(t) = \bar{v}^*(t)$$

$$\underline{u}^*(0) = \underline{u}^*(t) = \underline{v}^*(t)$$



EMPIRICAL FINDINGS

Introducción

- The empirical application of this methodology, in the context of our PhD thesis, consists on the estimation of the 2013 SUT with the same classification of industries and products observed in the 2008, 2011 and 2016 SUT.
- For the calculations, we consider:

$V^A = \textit{Supply Table 2013}$

$U^A = \textit{Use Table 2013}$

$V^W = \textit{Supply Table 2011}$

$U^W = \textit{Use Table 2011}$

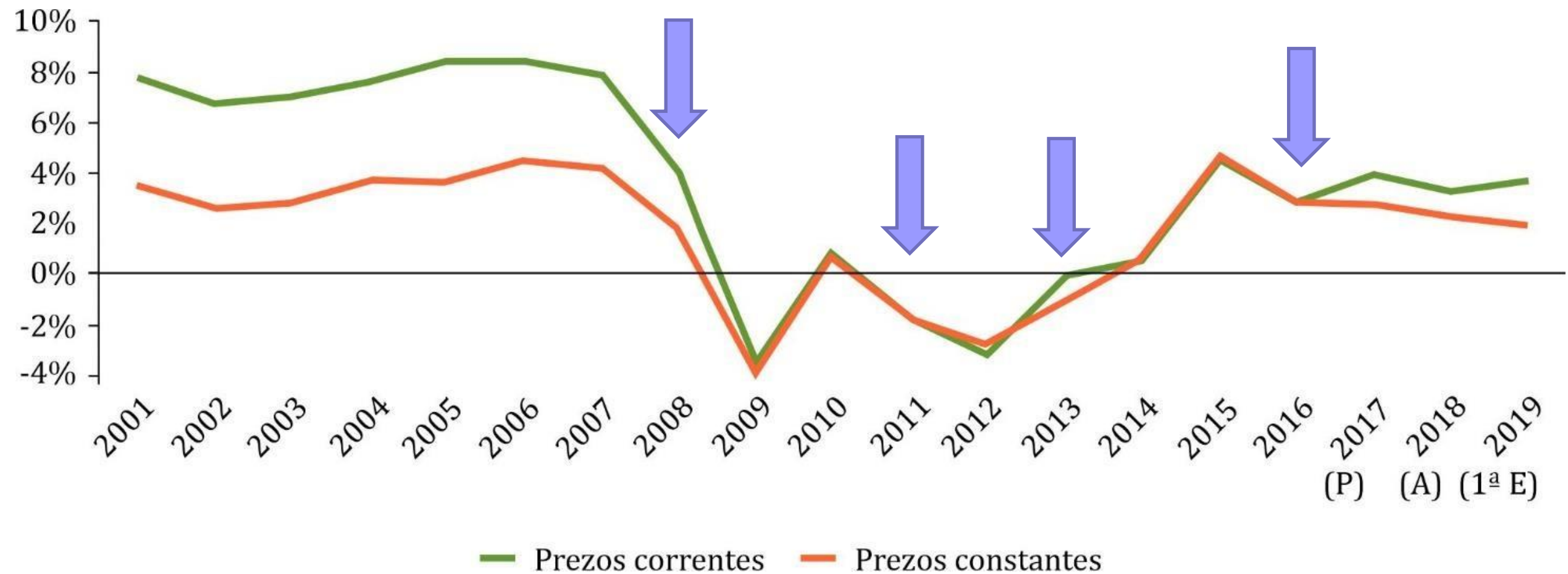
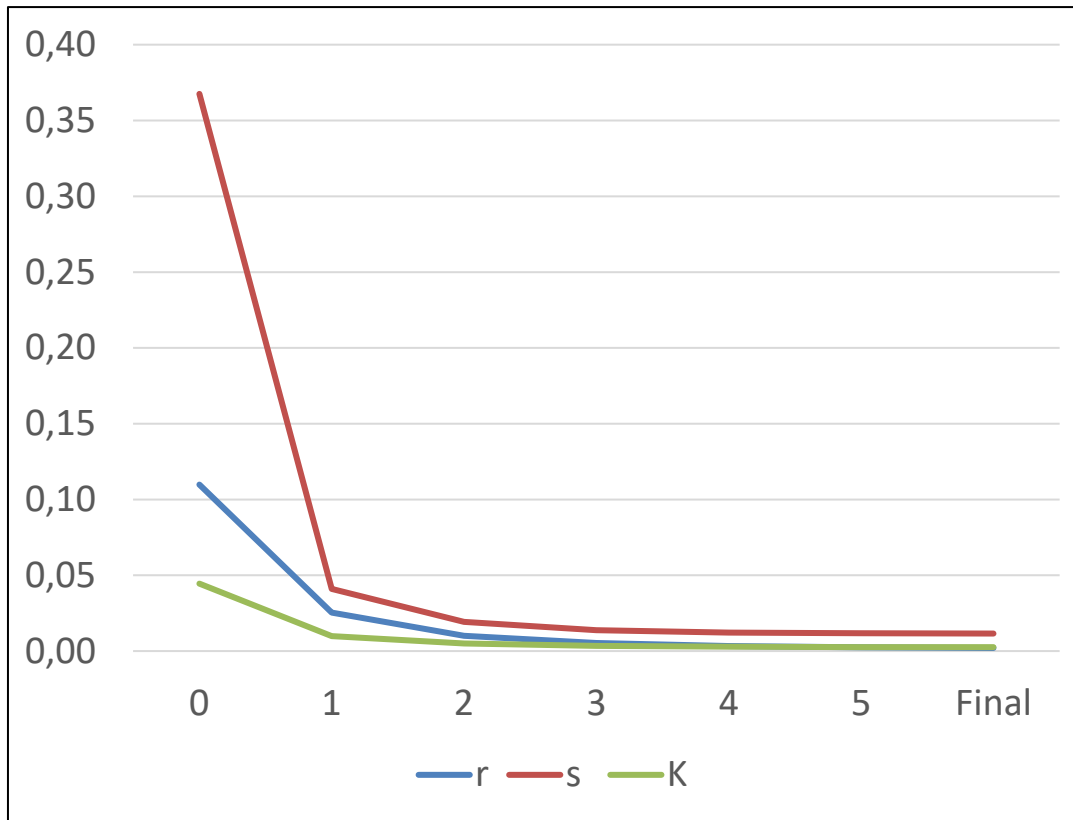


Gráfico 2. Taxas de variación anuais do PIB de Galicia a prezos correntes e a prezos constantes (euros de 2016). Anos 2001-2019. Fonte: elaboración propia a partir de INE, *Contabilidade Regional de España*.

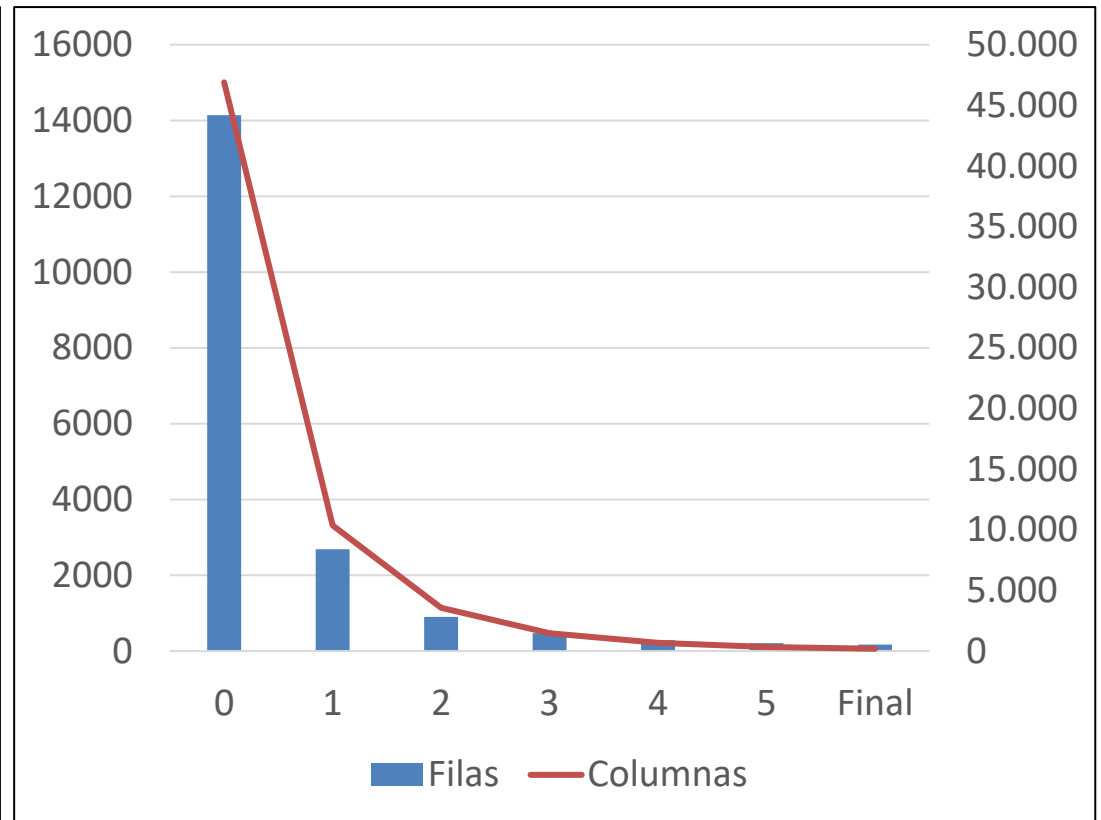


SUPPLY TABLE

Convergence towards a unique solution:



Mean absolute deviation from 1 of the r, s, and k coefficients in each iteration.

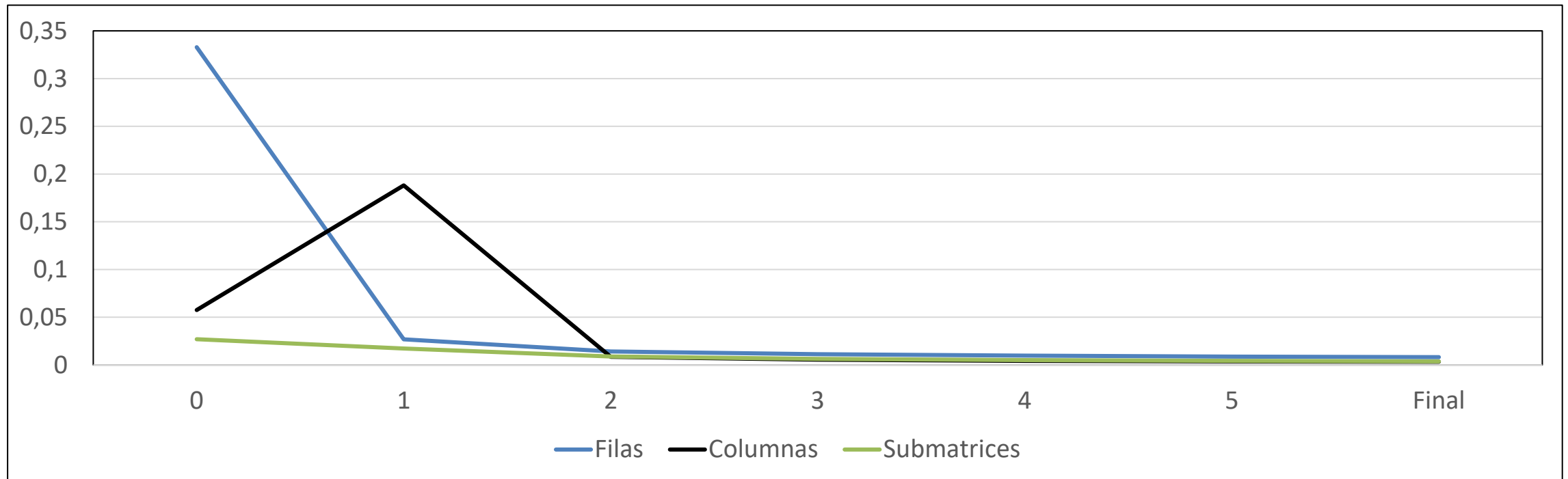


Mean absolute difference between the elements of $\bar{v}^*(\tau)$ and $\underline{v}^*(\tau)$ and their correspondents in the previous iteration.



USE TABLE

Convergence towards a unique solution:



Mean absolute deviation from 1 of the r, s, and k coefficients in each iteration.

Structural similarities between the 2011 and the 2013 Use tables

Higher technical coefficients			
Código	2013	Código	2011
R19	Coquerías e refino de petróleo	R19	Coquerías e refino de petróleo
R10C	Fabricación de produtos lácteos	R10C	Fabricación de produtos lácteos
R10D	Fabricación de produtos para a alimentación animal	R10D	Fabricación de produtos para a alimentación animal
R10A	Procesamento e conservación de carne e elaboración de produtos cárnicos	R37_38NM	Actividades de saneamento e xestión de residuos de non mercado
R37_38NM	Actividades de saneamento e xestión de residuos de non mercado	R10A	Procesamento e conservación de carne e elaboración de produtos cárnicos
Lower Technical coefficients			
Código	2013	Código	2011
R74_75	Outras actividades profesionais, científicas, técnicas e veterinarias	R72	Investigación e desenvolvemento
R68	Actividades inmobiliarias	R68	Actividades inmobiliarias
R78	Actividades relacionadas co emprego	R78	Actividades relacionadas co emprego
R85NM	Educación de non mercado	R85NM	Educación de non mercado
R97	Actividades dos fogares como empregadores de persoal doméstico	R97	Actividades dos fogares como empregadores de persoal doméstico



Work in progress

DISAGGREGATING THE 2016 PUBLISHED TABLES

Accuracy of the estimation: an example taken from the Supply Table

		R10A	R10B	R10C	R10D	R10E	R11	R12
		Procesamento e conservación de carne e elaboración de produtos cárnicos	Procesamento e conservación de peixes, crustáceos e moluscos	Fabricación de produtos lácteos	Fabricación de produtos para a alimentación animal	Outras industrias alimentarias	Fabricación de bebidas	Industria do tabaco
10A1	Carne elaborada e en conserva	1,13	1,00	1,00	1,00	1,00	1,00	1,00
10A2	Produtos cárnicos	1,13	1,00	1,00	1,00	1,00	1,00	1,00
10B1	Conxelados (pescado, moluscos e crustáceos)	1,00	0,75	1,00	1,00	1,00	1,00	1,00
10B2	Conservas (pescado, moluscos e crustáceos)	1,00	0,93	1,00	1,00	1,00	1,00	1,00
10B3	Outros preparados a base de pescados, crustáceos e moluscos	1,00	0,98	1,00	1,00	1,00	1,00	1,00
10C1	Leite de consumo líquida e en pó	1,00	1,00	1,44	1,00	1,00	1,00	1,00
10C2	Derivados lácteos e xeados	1,00	1,00	1,14	1,00	1,00	1,00	1,00
10D	Produtos para a alimentación animal	1,00	1,00	1,00	1,12	1,00	1,00	1,00
10E1	Froitas e hortalizas, preparadas e en conserva	1,00	1,00	1,00	1,00	0,55	1,00	1,00
10E2	Aceites e graxas vexetais e animais	1,00	1,00	1,00	1,00	0,61	1,00	1,00
10E3	Produtos do muíño, amidóns e amiláceos	1,00	1,00	1,00	1,00	0,82	1,00	1,00
10E4	Outros produtos alimenticios	1,00	1,03	1,00	1,00	0,86	1,00	1,00

Ratios between our 2016 estimation and the values observed in the published Supply Table.



CONCLUSIONS AND FURTHER INVESTIGATION POSSIBILITIES

Conclusions: strengths of the “GT-SUT-RAS”

- The methodology described allows the disaggregation of SUTs incorporating all the information available about the technology of the new industries or productions. Note that this methodology can incorporate more information beyond the data collected in the reference matrices.
- GT-SUT-RAS yields unique and non-probabilistic solutions for the construction of the new SUTs. It also ensures the coherence and economic meaning of the matrices obtained.
- GT-SUT-RAS is coherent with the available literature both for disaggregation of Input-Output Tables and IPF procedures. Most of the mechanisms used have already been academically tested.
- Finally, GT-SUT-RAS is easy to handle. As an example of this, the empirical application here presented was entirely elaborated with Excel. Basic skills on using Visual Basics allow to automatise most of the calculations.

Further investigation possibilities

■ Work in progress:

- For handling contradictory and irreconcilable information, it seems necessary to find a procedure for the calculation of optimal α_i and β_j .
- More systematization is needed too when solving the problems derived from the great proportion of non-adjustable elements in the matrices, namely in the Supply Table.

■ Other application possibilities:

- These type of methodologies could be used to elaborate coherent MRIO models in contexts of limited information.
- Other possible uses could be environmental or employment analysis.

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Discussion

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