Inforum before Inforum

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In this, likely to be my last, contribution to an Inforum conference, it may be of some interest to look back to Inforum before its birth, to Inforum in the womb of the Harvard Economic Research Project (HERP) better and more properly known as the Leontief Project. When I joined the HERP staff as a graduate assistant in the fall of 1959 after two years in the Army I was given a large manual for programming the Univac I in machine language. It had been the first commercial computer and was still Harvard's main computer – though MIT had an IBM machine with Fortran. I have always been glad to understand what is going on in computers at this very basic level, but I was also relieved to learn that we had available a package of matrix routines.

Leontief was at that time trying to get a sensible solution to his dynamic system described a few years earlier in *Studies in the Structure of the American Economy.* I was soon involved in this effort. The dynamic system can be written

(1) $x(t) = Ax(t) + B\dot{x}(t) + f(t)$

where x(t) is a vector of outputs of various products, $\dot{x}(t)$ is its derivative with respect to t, time, f(t) is a vector of final demands not including products for expansion investment, and A and B are the usual input-output matrices for intermediate products and expansion investment, respectively.

Ask a mathematician how to solve this equation numerically given an initial value of x and he will almost certainly say, "Solve for $\dot{x}$ and then use any standard numerical method for solving differential equations." That was what Leontief was trying to do. The first problem was that B was highly singular with a number of all-zero rows. So we aggregated it up so that it had no all-zero rows. It then proved non-singular and we could calculate B^{-1}. But when (1) was solved for $\dot{x}$, one or more elements were invariably negative, and Leontief recognized that (1) made no sense for negative elements of $\dot{x}$. Investment processes cannot be run backwards turning machinery into the materials and labor required to make it. So we would that element of $\dot{x}$ equal to 0, and eliminate its equation. But then another element would turn negative. Leontief referred to this behavior as the "switching problem." He more or less gave up on the dynamic model and worked on other things.

I was determined to get a solution for my Ph.D. thesis. Halfway through the academic year I had allotted myself to write the thesis I had learned a lot about numerical solution of differential equations, Lagrangian interpolation, and other topics. But I had made no headway with the basic problem. One evening I sat down on the sofa in our apartment and said to myself, "If f(t) is a polynomial in t, then there is a particular solution of the same form. Suppose $f(t) = f_0 + f_1t$, then there is a particular solution of the form $q(t) = q_0 + q_1t$. Substituting into (1) gives

(2) $q_0 + q_1t = A(q_0 + q_1t) + Bq_1 + f_0 + f_1t.$

Equating the coefficients of the same powers of t, we have

(3) $q_0 = A\,q_0 + Bq_1 + f_0$

(4) $q_1 = Aq_1 + f_1.$

Equation (4) is readily solved $q_1 = (I - A)^{-1}f_1$ and (3) is then solved for q_0. The only matrix inversion required is that of (I - A) and that is well known to be free of problems and to give an $(I - A)^{-1}$ without negative elements. f_1 is normally non-negative and then so is q_1. Thus, we get perfectly sensible particular solutions. I quickly made the necessary calculations for a ten sector model.

Should we worry about probably quite wild terms of the general solution? I argued we need not worry about them because equation (1) is good only for determining x given $\dot{x}$, not for determining $\dot{x}$ given x. There is simply no economic mechanism that makes the various industries adjust their rates of growth to exactly use up a given vector of investment goods. If the resulting q(0) should come out far from the observed q(0), I argued we could add or subtract from the investment. Leontief wrote on my thesis at this point, "You have cut the Gordian knot, not untied it." In fact, no such adjustments were ever made.

The ten-sector model was the basis of my thesis, and what I had learned about Lagrangian polynomials spilled over into a suggestion to Shirley, my wife, for estimating the distributed lag between capital appropriations and investment. Her article on this subject became a classic. We shared so much that we agreed not to write sentimental acknowledgments to one another.

I was appointed an instructor and then an assistant professor at Harvard; and Shirley, an assistant professor at Wellesley. I was still committed to the idea of consistent forecasting, that is to the idea that the forecasts should be consistent with their being believed and investment decisions beimg based on them. But now we had the brand new 1958 input-output table, and Harvard had a new IBM computer with Fortran. I could write any sort of investment function I wanted. The details are in the appendix of *The American Economy to 1975.* Basically, the new program made initial projections of disposable income and investment, calculated consumption expenditures and then outputs. Then investment was recalculated and then outputs and that cycle repeated until investment and output were consistent. Then employment was calculated. If it was short of projected "full" employment, the assumed disposable income was revised upward and vice versa. The process turned out to converge quickly. Thus the forecasts were consistent with their being believed and with projected full employment. There was no calculation of personal income, taxes or government budgets.

Meanwhile, Shirley's work showed that even perfect forecasts of output were of no value in explaining corporate capital appropriations or expenditures. Firms normally operated with excess capacity, and only when the excess was reduced by growth in output did they invest in more capacity. When I moved to Maryland, I completely rewrote the forecasting program giving up the idea that investment should depend upon *forecasted* growth in output and instead making it depend upon replacement and recent *past* growth in output.

In 1966 I came to Maryland. The first models at Maryland still took disposable income as an exogenous variable which was manually adjusted to give a targeted level of employment. Only later was "The Accountant" part of the model added to make disposable income an endogenous variable and thus make it possible to run the model as a multisectoral business-cycle model. By adjusting the income tax rate in such a model to give a desired level of employment it is possible to return to the original idea of forecasts consistent with their being believed but now also consistent with the desired level of employment. Such forecasts went into 1*985: Interindustry Forecasts of the American Economy,* (D. C. Heath, 1974).

It is my impression that most Inforum models around the world today operate mainly as multisectoral business-cycle models, but it may be interesting to recall that their ancestors had the mission of showing the way to stable, full-employment growth.